

### 3.2 Higher Derivatives (Minimum Homework: 1, 3, 5, 9, 11, 15)

A **Second derivative** is just the derivative of a derivative.

The symbol  $f''(x)$  is used to represent the second derivative of the function  $f(x)$

A **Third derivative** is just the derivative of a second derivative.

The symbol  $f'''(x)$  is used to represent the third derivative of the function  $f(x)$

Here is a quick example:

$$\text{Given } f(x) = 3x^6$$

a) Find  $f'(x)$

b) Find  $f''(x)$ , the second derivative of  $f(x)$ .

c) Find  $f'''(x)$ , the third derivative of  $f(x)$ .

a)  $f'(x) = 6 * 3x^{6-1}$

$$f'(x) = 18x^5$$

b)  $f''(x) = 5 * 18x^{5-1}$

$$f''(x) = 90x^4$$

c)  $f'''(x) = 4 * 90x^{4-1} = 360x^3$

#1 – 6: For each function find the first second and third derivatives.  
Specifically find

a)  $f'(x)$

b)  $f''(x)$

c)  $f'''(x)$

1)  $f(x) = 12x^4 + 5x^3 + 3x^2 - 6x + 1$

2)  $f(x) = 3x^5 - 8x^4 + 4x^3 + 5x^2 + 3x + 5$

*answer* a)  $f'(x) = 15x^4 - 32x^3 + 12x^2 + 10x + 3$

b)  $f''(x) = 60x^3 - 96x^2 + 24x + 10$

c)  $f'''(x) = 180x^2 - 192x + 24$

$$3) f(x) = -3x^3 + 6x^2 + 8x + 9$$

$$4) f(x) = -2x^4 + 3x^3 + 4x^2 - 2x - 1$$

*answer:* a)  $f'(x) = -8x^3 + 9x^2 + 8x - 2$

b)  $f''(x) = -24x^2 + 18x + 8$

c)  $f'''(x) = -48x + 18$

$$5) f(x) = 3x^{-5}$$

$$6) f(x) = 2x^{-3}$$

$$\text{answer a) } f'(x) = -6x^{-4} = -\frac{6}{x^4}$$

$$\text{b) } f''(x) = 24x^{-5} = \frac{24}{x^5}$$

$$\text{c) } f'''(x) = -120x^{-6} = -\frac{120}{x^6}$$

#7 – 18: For each function find the first and second derivatives.  
Specifically find

a)  $f'(x)$

b)  $f''(x)$

7)  $f(x) = e^{2x+3}$

8)  $f(x) = e^{2x+5}$

9)  $f(x) = e^{x^2}$

10)  $f(x) = e^{x^3}$

Answer a)  $f'(x) = 3x^2e^{x^3}$    b)  $f''(x) = 3xe^{x^3}(3x^3 + 2)$

$$11) f(x) = \ln(2x)$$

$$12) f(x) = \ln(3x)$$

$$\text{Answer a) } f'(x) = \frac{1}{x} \quad b) f''(x) = -1/x^2$$

$$13) f(x) = \ln(x^2)$$

$$14) f(x) = \ln(x^3)$$

$$15) f(x) = \frac{x+2}{x-3}$$

$$16) f(x) = \frac{x+4}{x-5}$$

$$\text{answer: a) } f'(x) = \frac{-9}{(x-5)^2}$$

$$b) f''(x) = \frac{18}{(x-5)^3}$$

$$17) f(x) = \frac{2x}{x+8}$$

$$18) f(x) = \frac{3x}{x-5}$$